

Working the Angles

Understanding force vectors in tree climbing



Text and illustrations by Joe Harris

CONTENTS

Introduction	3
1 Basic Forces	4
1.1 Rope Angles and Forces	5
1.2 Features of the Angle Rule	6
1.3 Applications of the Angle Rule	7
1.4 The Force Rule; Some Examples	8
1.5 The Force Rule in Practice	9
1.6 Torque	10
2 Taking it Further	12
2.1 The Torque Law	13
2.2 The Cosine Law	15
2.3 Putting it All Together	16
3 More Applications of Force Vectors	17
3.1 Snatching	17
3.2 Pulling Trees Over	20
3.3 Double Crotching and High-Lining	21
4 Summary	23
5 Appendix I: Magnitude Graphs	24

INTRODUCTION



It is hard to spend any time climbing and working in trees without developing a good hands-on knowledge of the forces involved. Climbers quickly get a 'feel' for the strength and flex of anchor points, for the forces involved in rigging and for the way a good rope angle offers better support.

This document is an attempt to provide some very basic principles which can be used to underpin the working knowledge of climbers, and which will hopefully allow a better understanding of the forces involved in climbing and rigging.



Take a look at the three pictures on the left. The climber is rigging the left-hand head out of a *Corymbia citriodora*, towards the end of the removal. The climber is set in the central stem; the rigging line is set across **both of the other two stems**. If you already know why, then you already have a good working knowledge of force vectors.

The camera was moved slightly as these pictures were taken, so it is hard to see clearly, but the left-hand stem stays almost stationary, despite having the head snatched out of it. The setup was far from perfect, however: the right-hand stem was pulled sharply **upward**, which is visible in the photograph.



In the first section, we will look at some simple vector additions and commonplace work scenarios. The second part of this document will go into greater detail with the physics behind some of the things you do every day at work.

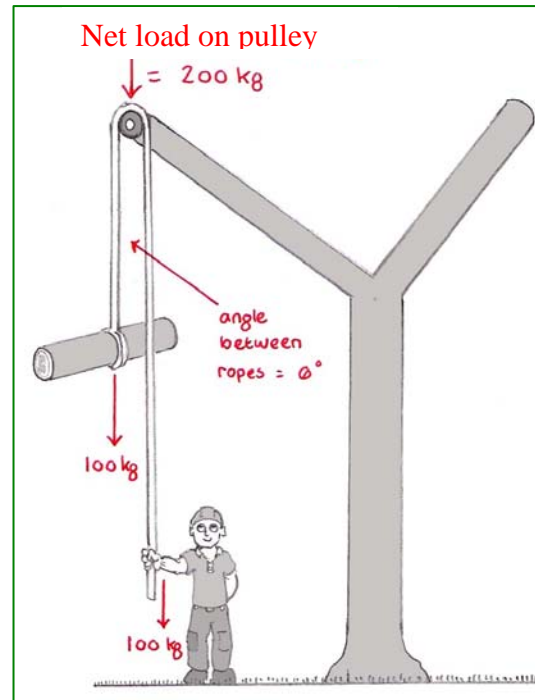
1 BASIC FORCES.

Take a look at the diagram on the right. It's a fairly common scenario in most removals: the climber has snatched off a large piece, and the groundie has brought it to a gentle stop with the lowering rope. Leaving aside for the moment any questions of shock loading, friction, and elastic elongation, how much load is on the pulley?

Almost any experienced climber will tell you straight away: "there's double the weight of the log." It seems fairly straightforward, but it's worth going into it in more depth.

The log weighs 100kg. As it is not plummeting to the earth something must be pulling in the other direction with the same force to keep it in place (Newton's Third Law). In this case it is clearly the rope which is holding the log in the air.

Over on the other side, the groundie is having to put his whole 100kg of weight on the rope to balance out the weight of the log.



It's fairly easy to see that in this simple example, where both sides of the rope are pulling down on the pulley with a weight of 100kg each, there is a **net load on the pulley of 200kg**.

To put it another way, when one side of a rope is loaded and the other is locked off,

Ropes running parallel = double the force on the anchor point

This simple statement is really important, and has a great many practical repercussions in tree climbing. The most common examples are rigging scenarios like the one shown above, and the classic Single Rope Technique setup of one side of the access line being secured to a ground anchor in order to make the other side ready for climbing.

"Shouldn't we be talking about forces instead of weights?"

In all of the examples in this section the loads are discussed purely in terms of weights. **This isn't strictly accurate**, as to work things out properly we should move across into Newtons and use genuine **force vectors**. This way is much simpler, however, and gives basically the same results.

Physicists may however wish to skip through to section 2, where we return to the correct use of forces to consider these scenarios in greater depth.

1.1 ROPE ANGLES AND FORCES

So far, so easy. As we saw on the previous page, lowering a section of timber whilst your groundie stands directly underneath causes a force on the pulley of double the weight of the piece being lowered. This is about as far as many climbers go with working out forces, and it is certainly far enough to remove just about any conceivable tree. Simply put your rigging point on the main stem somewhere up near the top of the tree, and then use it to lower off as large a bit as you think it can handle.

THE ANGLE RULE:

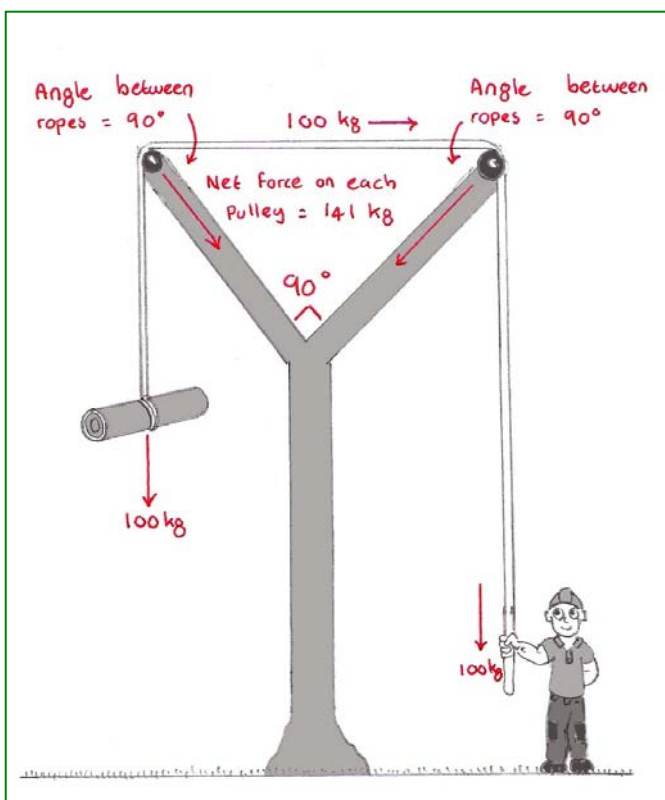
The resultant force of a rope passing over a point bisects the angle made by the rope at the point.

THE FORCE RULE:

The wider the angle made by a rope passing over a point, the less force the rope applies to that point.

But we can do better. **The Angle Rule**, given above right, is a fantastic and simple mental tool to carry in your head when thinking about the forces that your climbing and rigging are going to apply to the tree.

In the picture below, the same log is being held up by the same groundie. But the setup, and the forces being applied on the tree, have completely changed. By adding a second pulley into the system on the right-hand head the climber has hugely improved the situation. In fact, **the weight placed on the tree at each pulley is greatly reduced (The Force Rule)**, but we'll get to that in a minute. For now, the most important thing to look at in the picture is the **direction in which the weight is applied**.



As you can see, the lowering rope is making a right-angle at each pulley. A quick look at **The Angle Rule** above will remind you that where two forces are pulling at angles to each other, the **net resultant force bisects the angle between the two forces**. In other words, it pulls exactly half-way between the two of them.

Half of 90° is 45° , so the resultant force on the pulley acts at 45° from the line of either rope. In this case, that means it is acting **exactly along the branch**. Perfect compression force!

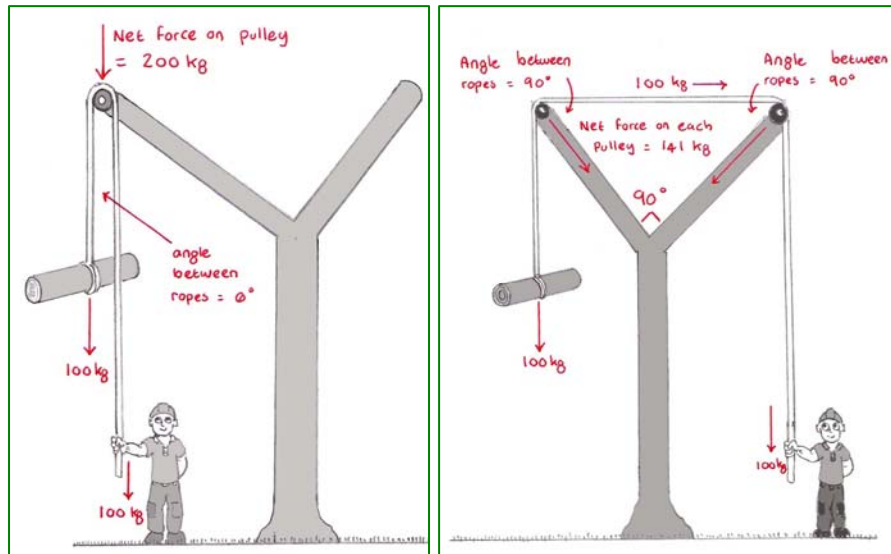
And another thing...

Some of you may already have spotted the other great advantage of this setup, which is that the groundie is no longer standing directly under the log being lowered.

1.2 FEATURES OF THE ANGLE RULE

An interesting property of the Angle Rule is that **no matter how many changes in direction the rope makes the force in the rope stays the same**. In the real world of course this isn't strictly true, as each bend in the rope adds friction, which reduces the amount of force required to hold it still. For the purposes of this discussion, however, try to keep in mind that by changing the angle that the rope makes as it passes over a pulley, **all we are doing is changing the force applied to that pulley**.

The interesting thing about this property of the rule is that **the net force applied to the tree is the same** no matter how many pulleys you put into it. Take another look at the two setups that we looked at before



If we treat the tree as a completely solid and static object (our example is becoming increasingly divorced from reality!), the tree in both cases has 200kg of downward weight being applied to it. The only differences are the angles at which it is being applied, and the point of application.

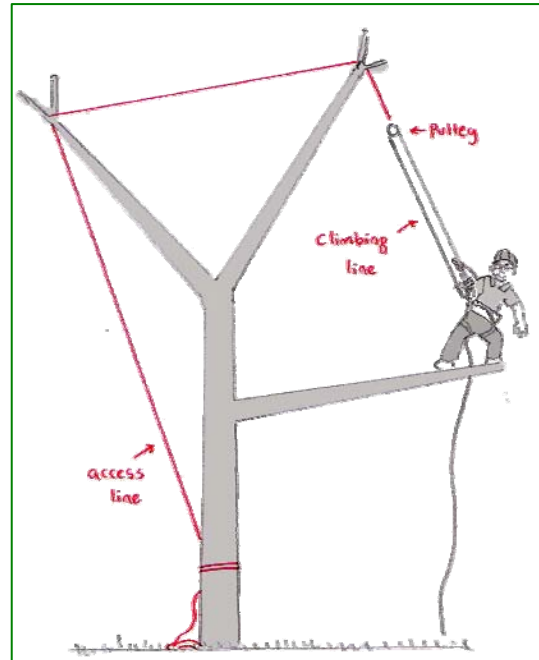
It is worth bearing in mind that whilst this comparison is useful for illustrative purposes, in reality the true nature of trees and in fact the vector of the forces being applied makes the above claim incorrect.

1.3 APPLICATIONS OF THE ANGLE RULE

The picture on the right shows a climber making good use of the Angle Rule. He wishes to climb out along the branch on which he is standing, and has no work to do anywhere else in the tree. Why is his setup so good?

Look at the force being applied at both of the forks which his access line passes through. In each case, the net force on the stem acts close to **directly along the stem**; in other words the net force on each stem is almost completely compression force.

Now imagine that the climber had simply set on the right-hand fork in the traditional manner. There would of course be no force applied at all on the left-hand stem. There would also be **less total force applied on the right hand stem**, as the climber would be applying only the single load of a normal setup. The important factor in this instance, and the reason why this setup works so well, is the **direction (vector) of the force**.



Imagine a tree similar to the one shown in the picture (maybe with a few leaves and some more branches!). Both *Corymbia citriodora* and *Eucalyptus cladocalyx* often present similar challenges to climbers, so for the sake of the example picture the tree as a spreading, skinny lemon-scented gum (a great example if you know it is the large *citriodora* in Deepdene, on the east side of Burke Rd., near the intersection with Whitehorse Rd.). Think of that right-hand stem as a long smooth rising leader, branching at the top into sparse foliage.

Now imagine the way in which it would move if the climber was pulling sideways on it using a traditional setup. It would flex and bend alarmingly to the right.

Using this setup, the stem would be moved about a bit, but (so long as there wasn't too much friction where the access line passes through the fork) the whole setup would feel very reliable and secure.

Warning

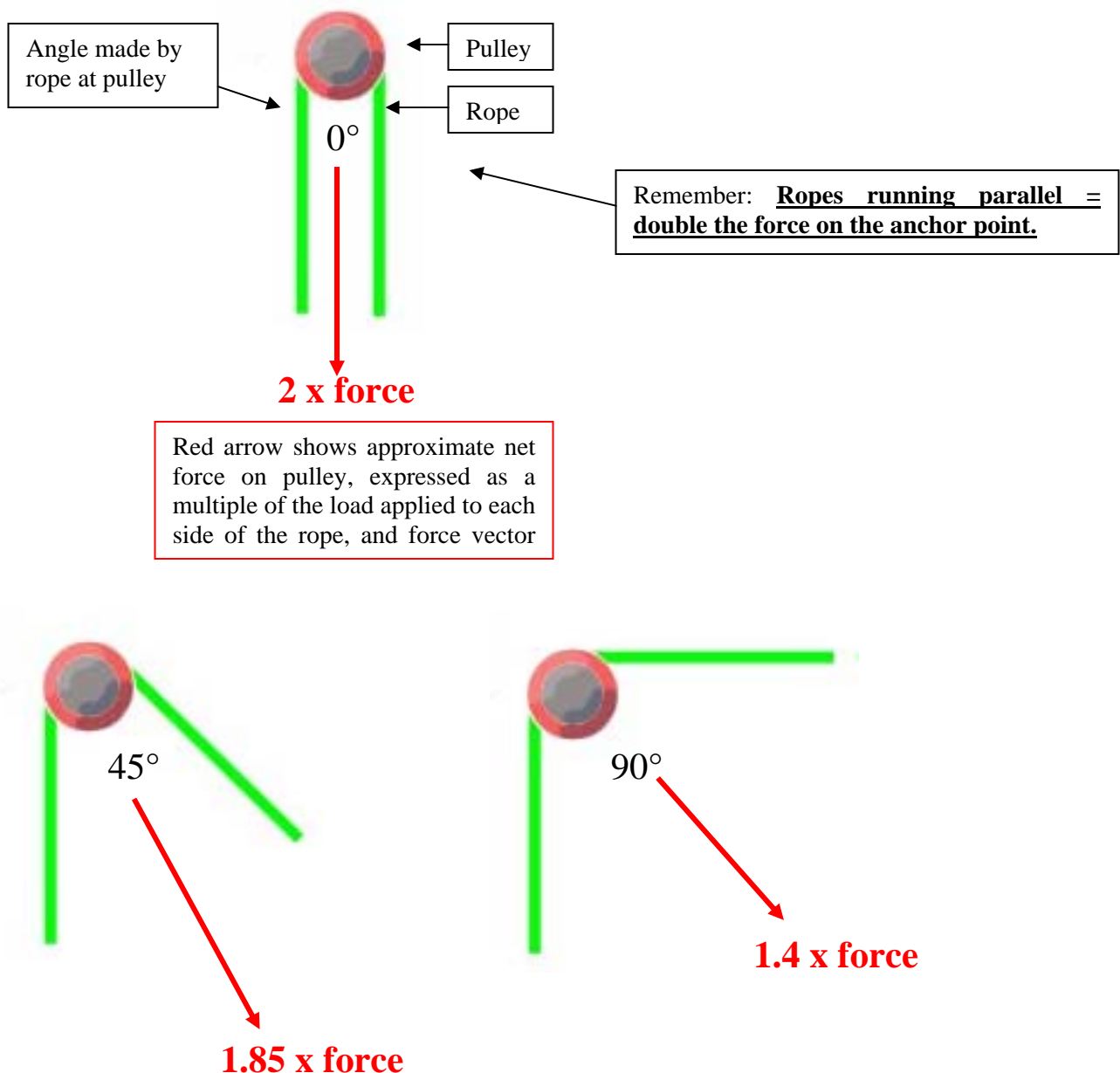
The setup shown in the diagram above is extremely strong **only for the direction in which the climber is currently heading**.

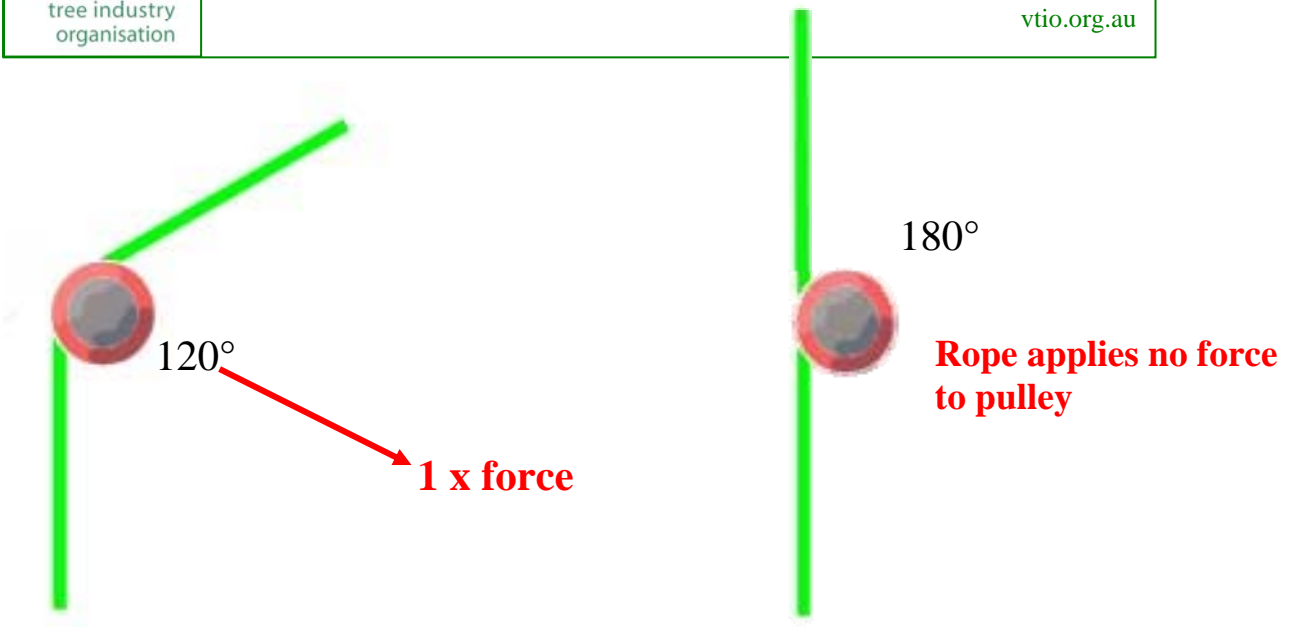
If he decides to go up and prune a bit out of the left-hand head, **this setup becomes extremely weak**. Imagine the force that would be applied to the right hand fork once he reached the left hand head, particularly if he fell through a fork of that head. Remember **Ropes running parallel = double the force on the anchor point**.

1.4 THE FORCE RULE; SOME EXAMPLES

THE FORCE RULE:

The wider the angle made by a rope passing over a point, the less force the rope applies to that point.

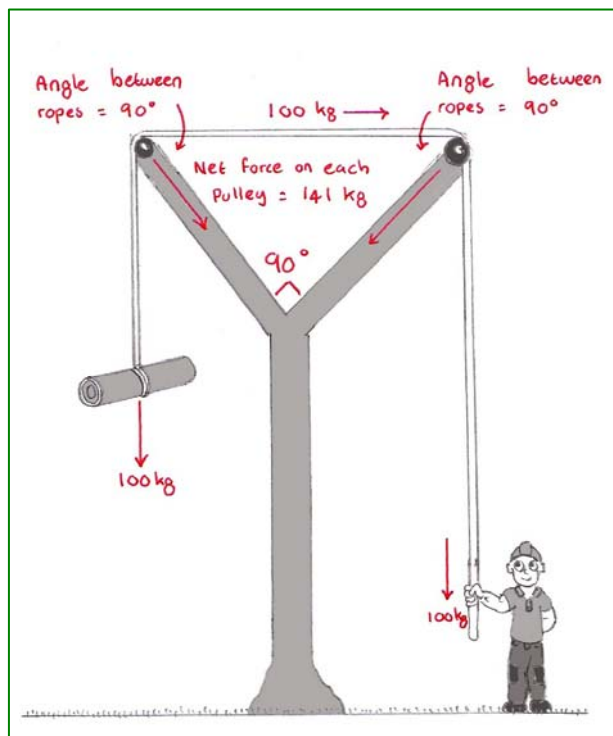




1.5 THE FORCE RULE IN PRACTICE

Let us return again to the picture shown on the right. We saw how **The Angle Rule** meant that the force on each pulley acted exactly along the branch. We can now use **The Force Rule** to work out that the wider the angle made, the less force applied. And in the picture to the right, because the angles match up neatly with one of the examples above, we know approximately how much force is actually being applied at each pulley.

To work out the forces produced by a rope angle that has not been given above, you can use the graph given in **Appendix I: Magnitude Graphs** on page 24.



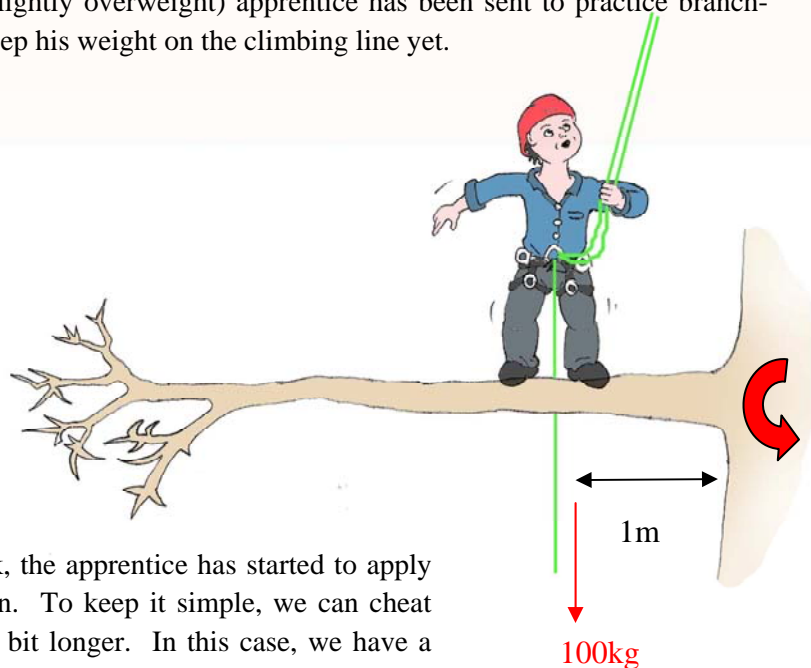
1.6 TORQUE

Torque, also called **turning moment**, is the tendency of a force to rotate an object about an axis.

TORQUE RULE:
TORQUE = FORCE X DISTANCE.

The most common use for arborists is probably using a lever or log roller to move a fallen trunk around – **the force you apply is multiplied by the length of lever arm you are using.**

It's a really difficult rule to use when thinking about the effects of forces on trees, as trees take it into account when growing (See Claus Mattheck's *Axiom of Uniform Stress*) and then flex to compensate for it (Mattheck's *Strategy of Flexibility*). We can think about it in general terms however, as we consider some different possible climbing and rigging situations. Take a look at the two pictures below. The (slightly overweight) apprentice has been sent to practice branch-walking, but hasn't learned to keep his weight on the climbing line yet.



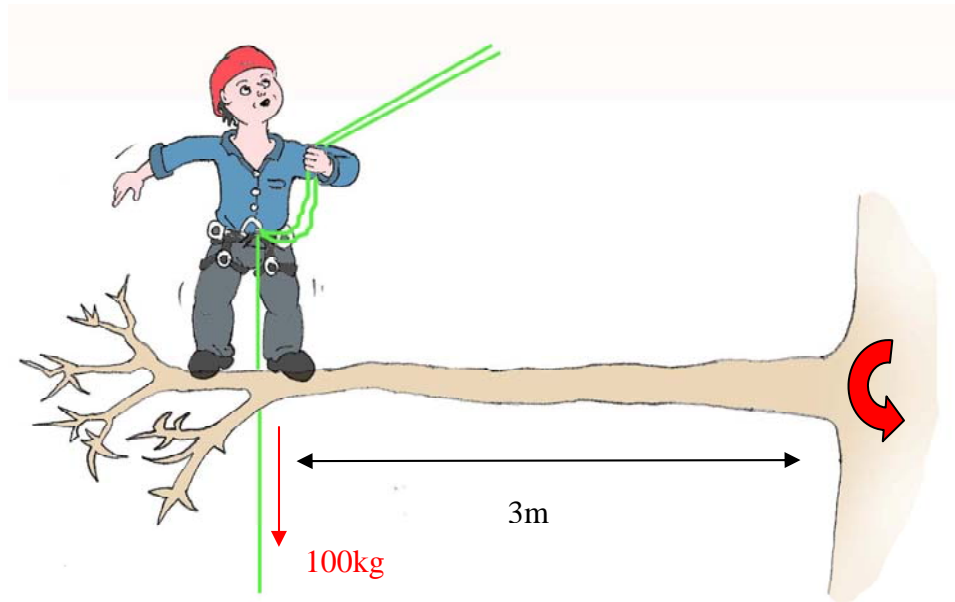
At one metre out from the trunk, the apprentice has started to apply some torque to the branch union. To keep it simple, we can cheat and stay in kilograms for just a bit longer. In this case, we have a 100 kilo apprentice 1 metre out from the trunk:

Torque = Force x Distance

Torque = 100 x 1

Torque = 100

In actual fact, this number has no real meaning, but for the sake of the argument we can say that the branch union is experiencing an extra 100kg of twist because of the apprentice.



He's made it all the way to the end! He probably hasn't gained any weight during this exercise (in fact maybe the opposite) so he still weighs 100kg. But his distance from the trunk has trebled:

Torque = Force x Distance

Torque = 100 x 3

Torque = 300

Pretty simple stuff. He's three times further away from the trunk, so he's exerting three times as much torque on the branch union.

As previously mentioned, the application of this rule to the real world of trees is fraught with difficulties. Trees rarely grow neat flat branches of perfectly even thickness, as they would have to for this rule to be of much use, and in any case they flex under load. These basic examples have been included mainly as background for the more complicated (and useful) version of the rule included in the next section.

2 TAKING IT FURTHER

In the previous section, we looked at some basic forces, and considered their practical applications in tree work. There is no need to read any further! This section will look at the mathematical principles which underpin the scenarios given previously, and is intended to provide the tools for climbers to apply these principles in a greater range of situations than the limited scope offered in this document. As we all know, it can be prohibitively difficult to accurately model the real world, which includes all of the factors such as friction; shock load; elastic elongation; branch flex and taper which have been deliberately excluded from our worked examples. In addition, climbers are almost always dealing with unknown or hard-to-know working loads, which makes the use of mathematics and modelling a somewhat futile exercise.

So why bother? The main reason is that an understanding of the physics behind a lot of what we do in the practical world gives the opportunity to understand and think about things in much greater depth. Particularly when you have a potentially difficult practical problem ahead of you, understanding the principles behind the rules given in the previous section may allow you to work out which of several possible options generates the best force on a tree, or provides the most useful force to help you in your work.

WORKING IN NEWTONS

As we go deeper into the principles behind our rules, it will no longer be good enough to ‘fudge’ the equations by using kilograms. Tree workers uncomfortable with Newtons as a unit of force may like to remember the story that the root of all of Isaac Newton’s laws of motion was an apple falling on his head as he sat under an apple tree...

NEWTONS

One Newton is the amount of net force required to accelerate a 1kg mass by 1m/s^2

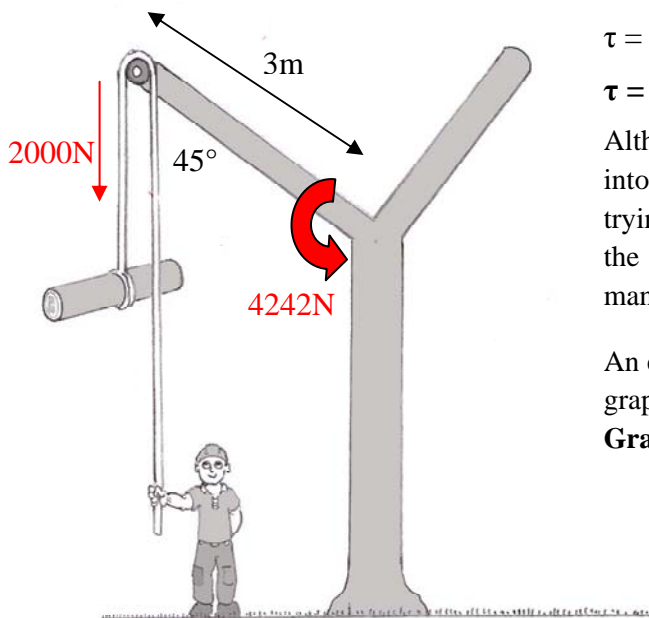
In this document, gravity has been approximated as 10m/s^2 , therefore a 1kg mass exerts a downward force of 10 Newtons.

2.1 THE TORQUE LAW

The **Torque Law**, given on the right, can seem worryingly mathematical at first. But it's almost the same as the simple **Torque Rule** we were using before. It is still Force x Distance, only now we are working in the angle between the force and the branch.

It is easy to use in practice and very useful.

Consider the picture shown below. We know that the force on a point when the two sides of the rope run parallel away from it is twice the load applied on either end of the rope. In this instance, that means that there is a force of 2000 Newtons pulling straight down on the stem where the pulley is attached. The distance between the stem and the pulley is 3 metres, and the angle between the stem and the force is 45°.



THE TORQUE LAW:

$$\tau = r F \sin\theta$$

τ is the torque applied at the axis

r is the distance from the axis to the force

F is the magnitude of the force being applied

θ is the angle between the force and the arm

Let's put those numbers into the equation:

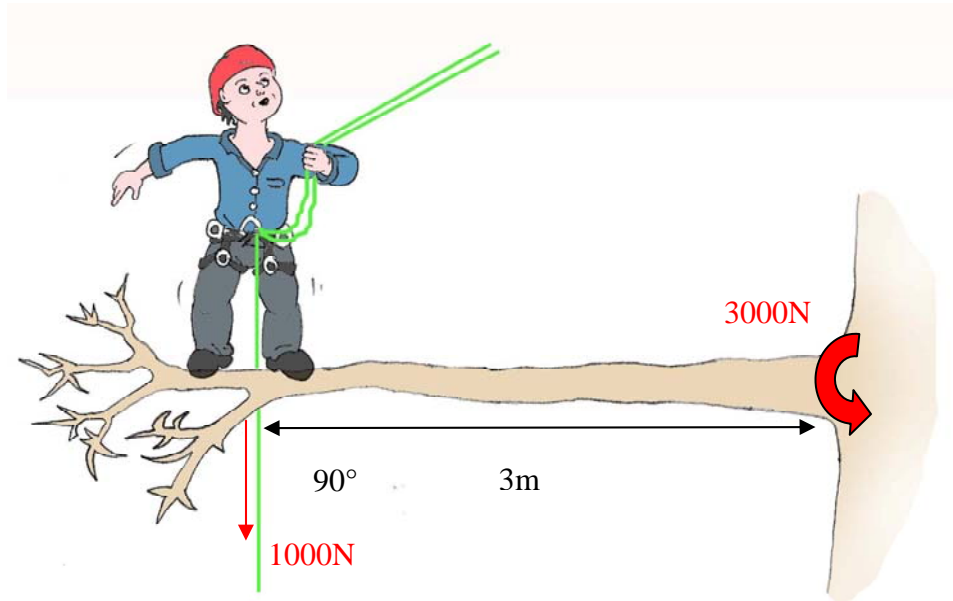
$$\tau = r F \sin\theta$$

$$\tau = 3 \times 2000 \times \sin 45^\circ$$

$$\tau = 4242 \text{ Nm}$$

Although it isn't really possible to swap back into kilograms, that is roughly the same as trying to break the branch off by twisting it at the union with enough force to lift a 424 kilo man!

An easy way to work with Torque is to use the graph given in **Appendix I: Magnitude Graphs** on page 25.



Let's have another look at the apprentice. He's still out there on the end of his branch, still weighing in at 100 kilograms, still not succeeding in putting any weight on his climbing ropes. The situation is actually much worse here than in the last example, because the force that the apprentice is applying has a vector exactly perpendicular to the angle of the branch.

$$\tau = r F \sin\theta$$

$$\tau = 3 \times 1000 \times \sin 90^\circ$$

$$\tau = \mathbf{3000 \text{ Nm}}$$

Another hard-to-use number, but it is essentially exactly the same as the '300kg of twist' we worked out before.

If we leave aside the *Strategy of Flexibility*, this is in fact a reasonably accurate tool to model the forces generated **at any point along the branch**. Simply measure the distance along the branch to the point that the force is being applied and put that into the equation. Clearly, the greater the distance the larger the torque. But this doesn't mean that the union is always the weakest spot!

Remember the *Axiom of Uniform Stress*, which accounts for the taper of the branch. A graph of torque along a branch or stem when load was applied near the tip would show that torque was directly proportional to distance... but in an idealised tree, we would find that branch or stem thickness was also directly proportional to distance. Leaving aside any factors such as cavities or the presence of decay organisms, the **weakest point will be where the ratio of torque to thickness of wood** (in direction of torque) **is highest**. This might well not be at the union.

2.2 THE COSINE LAW

THE COSINE LAW:

The net force **F** on a point will bisect the angle made by the rope at that point, such that:

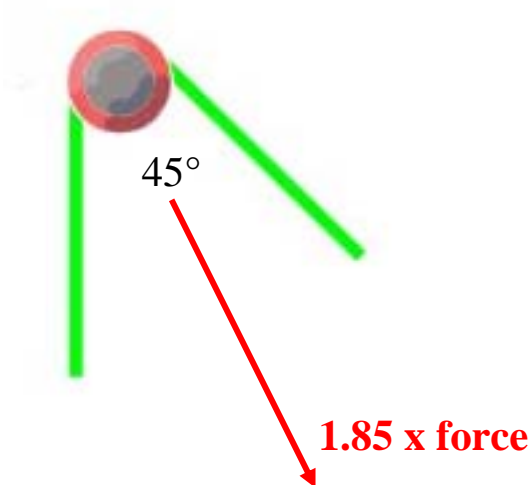
$$F^2 = 2X^2 + 2X^2\cos a$$

Where **X** is the force on the rope, and **a** is the angle made by the rope across the point.

The Cosine Law, given to the left, is another alarmingly mathematical concept to try and involve in the essential question of how to snatch the head out of an enormous mountain ash.

In practice there is no need to try and work out the equation, particularly because (as discussed previously) accurate modelling of tree dynamics is generally unfeasible.

Before we go any further, however, let us look at a couple of the **force rule** examples given previously, and use **the cosine law** to check that the force vectors shown in the diagrams are accurate. In both of the examples the rope is tied off at one end and holding a weight of 100kg on the other:



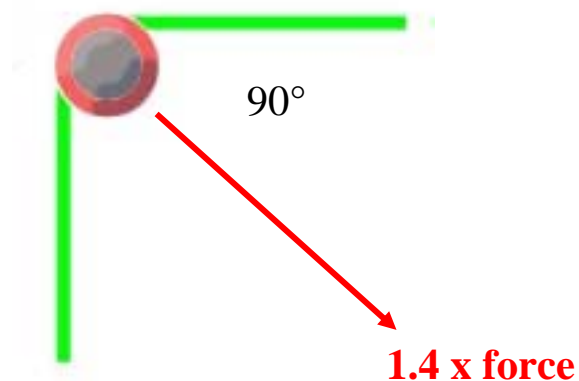
$$F^2 = 2X^2 + 2X^2\cos a$$

$$F^2 = 2(1000)^2 + 2(1000)^2\cos 45$$

$$F = \sqrt{(2,000,000 + 1414213)}$$

$$F = 1848 \text{ N}$$

$$F = 1.85 \text{ x Force}$$



$$F^2 = 2X^2 + 2X^2\cos a$$

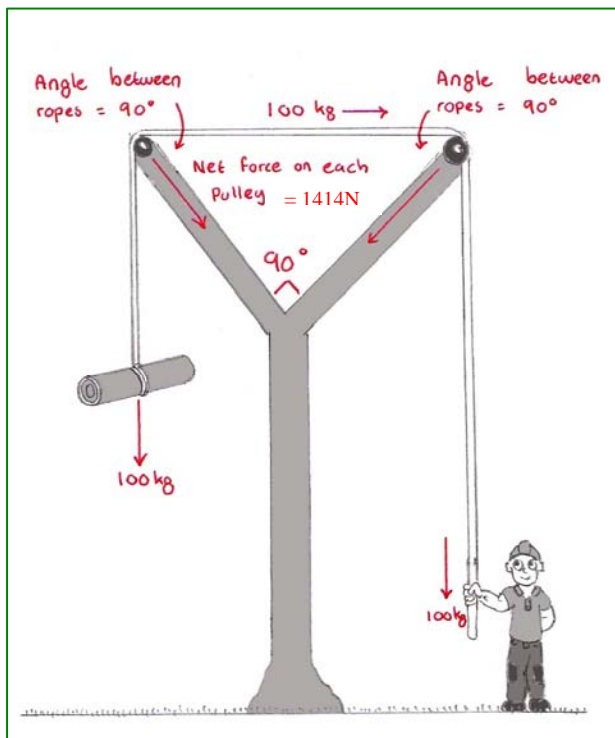
$$F^2 = 2(1000)^2 + 2(1000)^2\cos 90$$

$$F = \sqrt{(2,000,000 + 0)}$$

$$F = 1414 \text{ N}$$

$$F = 1.4 \text{ x Force}$$

2.3 PUTTING IT ALL TOGETHER



$$\tau = r \sqrt{(2F^2 + 2F^2 \cos a) \sin \theta}$$

This alarming equation is just a small part of the physics at work when you lower off a piece of wood. The full evaluation would need to include friction; shock loading; elastic elongation; groundie skill; the taper and flex of the branch. Plenty of time for the foreman to chip in with:

“Ah, just get out there and cut it off!”

Let us return for the last time to the picture shown on the left.

We have already used **The Cosine Law** to work out that the force applied to each pulley is **1.4 x the force along the rope**.

We can also use **The Torque Law** to work out that **there is no torque being applied at the union**: the force generated is acting as perfect compression force along each branch.

This is clearly an incredibly powerful mental tool to use in tree work, that is applicable to a wide variety of situations. In addition to the basic examples shown here, check out some more applications in the next section.

But is it worth learning?

Almost certainly not, unless you have a gift for mental arithmetic, are really good at estimating log weights and take your job very, very seriously.

Warning

In the example shown above, we worked out that there was **no torque being applied at the branch union**. But that does not mean that there is no force: there is still a great deal of compression force being applied to the branch. And because it is not being pulled to one side it can be much harder to judge the magnitude of the forces involved. **With sufficient force, the branch may suddenly collapse**, or a slight bend in the branch may cause it to bow to one side, and then split.

3 MORE APPLICATIONS OF FORCE VECTORS

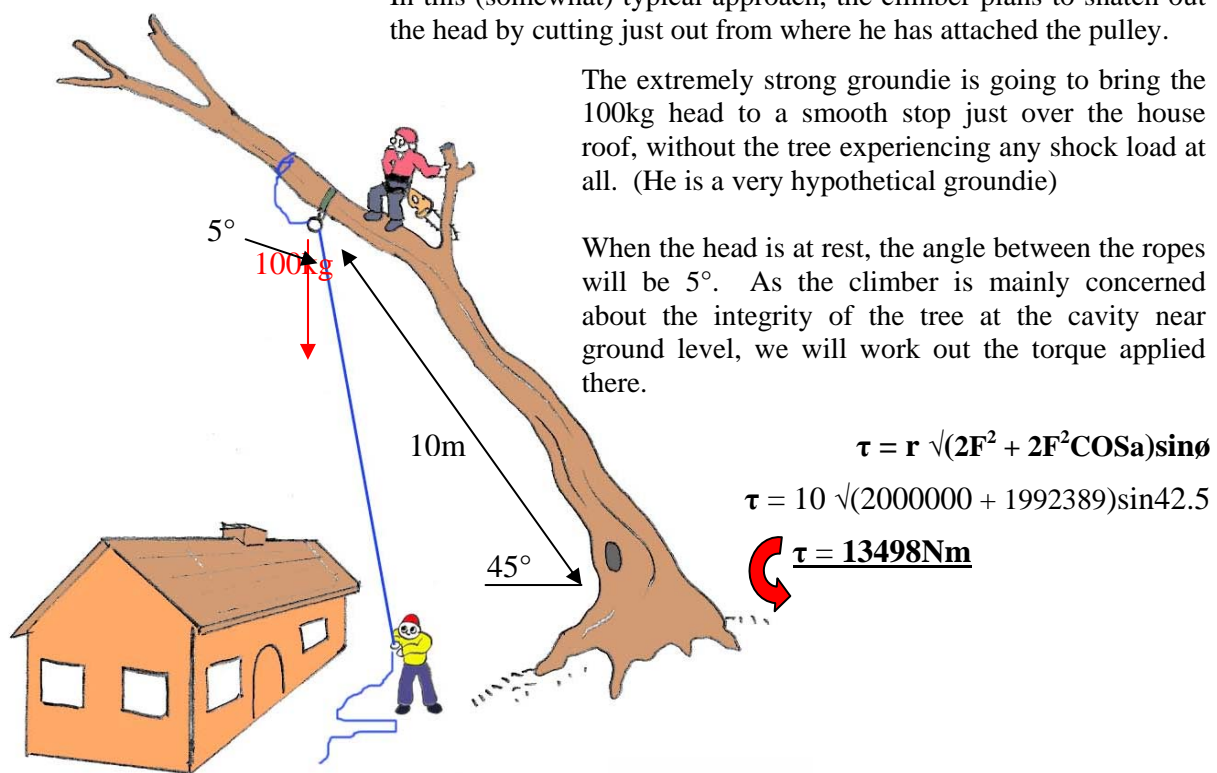
3.1 SNATCHING

In the series of pictures shown below, the climber is looking to lower out the end of the stem he is standing on. He isn't prepared to climb any further out along the stem, and he is concerned about the integrity of the tree near ground level. Although in this instance we are looking at the effect on an entire tree, in fact the same technique would work on any single branch.

In this (somewhat) typical approach, the climber plans to snatch out the head by cutting just out from where he has attached the pulley.

The extremely strong groundie is going to bring the 100kg head to a smooth stop just over the house roof, without the tree experiencing any shock load at all. (He is a very hypothetical groundie)

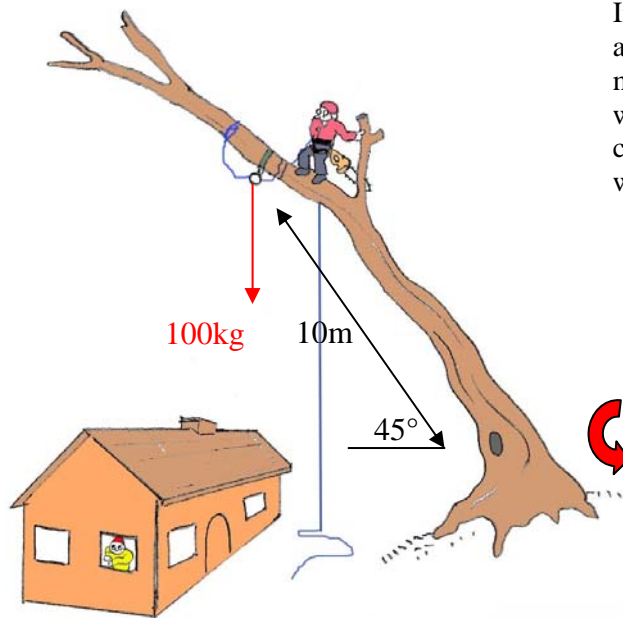
When the head is at rest, the angle between the ropes will be 5°. As the climber is mainly concerned about the integrity of the tree at the cavity near ground level, we will work out the torque applied there.



If that number seems quite abstract, it could also be phrased as the tree being twisted at the cavity with enough force to lift a 1.3 tonne ute into the air.

Some (often “old school”) arborists like to add friction at the lowering point, for example by taking a turn of the rope around a branch. This method has several drawbacks, but is included here as (when utilised by the climber wrapping friction and controlling the rope) it is an effective way of reducing force on a lowering point.

The main drawback is that snatching with wraps around the point of lowering is a risky business, as it requires a hand free to control the rope (we all know what that means) and is very hard to fine tune in terms of applying the right amount of friction to smoothly bring a heavy piece to a stop. In addition, multiple wraps may bind on each other as the rope is running.



In the scenario we are considering, let us assume that our experienced climber manages to bring the branch to a stop without experiencing any shock load or cutting himself whilst operating his saw with one hand.

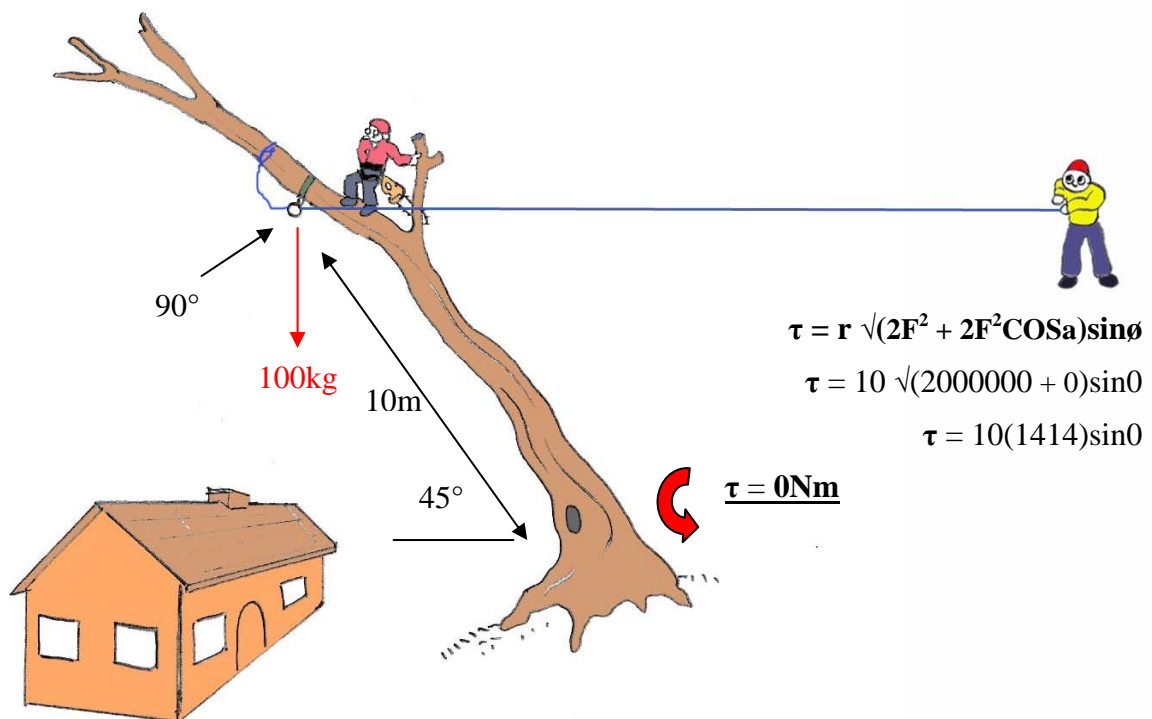
$$\tau = r F \sin\theta$$

$$\tau = 10 (1000) \sin 45$$

$$\tau = \underline{7071Nm}$$

In terms of the torque applied at the base of the tree, we have in fact made a huge improvement. As discussed above, the drawbacks of this method are not to do with the force generated – rather that the actual practice can be very risky. In addition, whilst in this circumstance it could be argued that it provides a safer outcome, in most situations under consideration a rigging point will be used for more than one cut, and wrapping friction on to a branch at the top of the tree before continuing to work lower down would not be an option.

In this last variation, the climber has asked his groundie to walk up the slope to the right, until he is level with the climber. **By walking around, the groundie is able to change the angle which the rope makes as it passes through the rigging point, and hence influence the force applied at that point.**

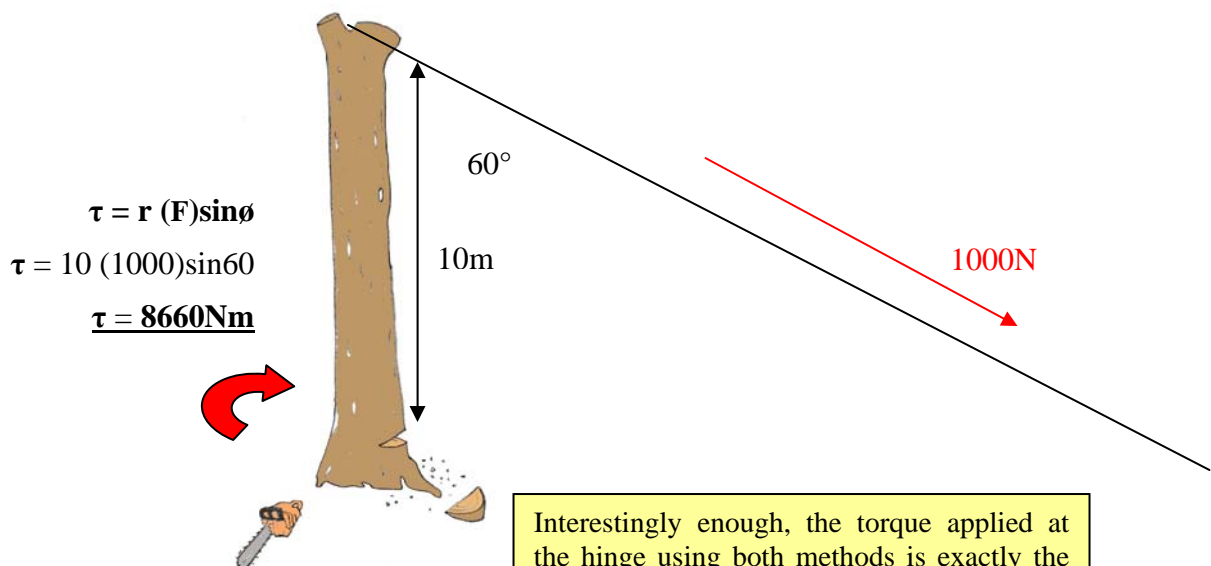


As we expected, we have reduced the turning moment on the tree to nothing. The interesting thing is that we are still working with more force in this example than the ‘old school’ climber shown on the previous page, so we still have to worry about the risks of compression failure or collapse, or the effects at various points in the tree where it bends or twists.

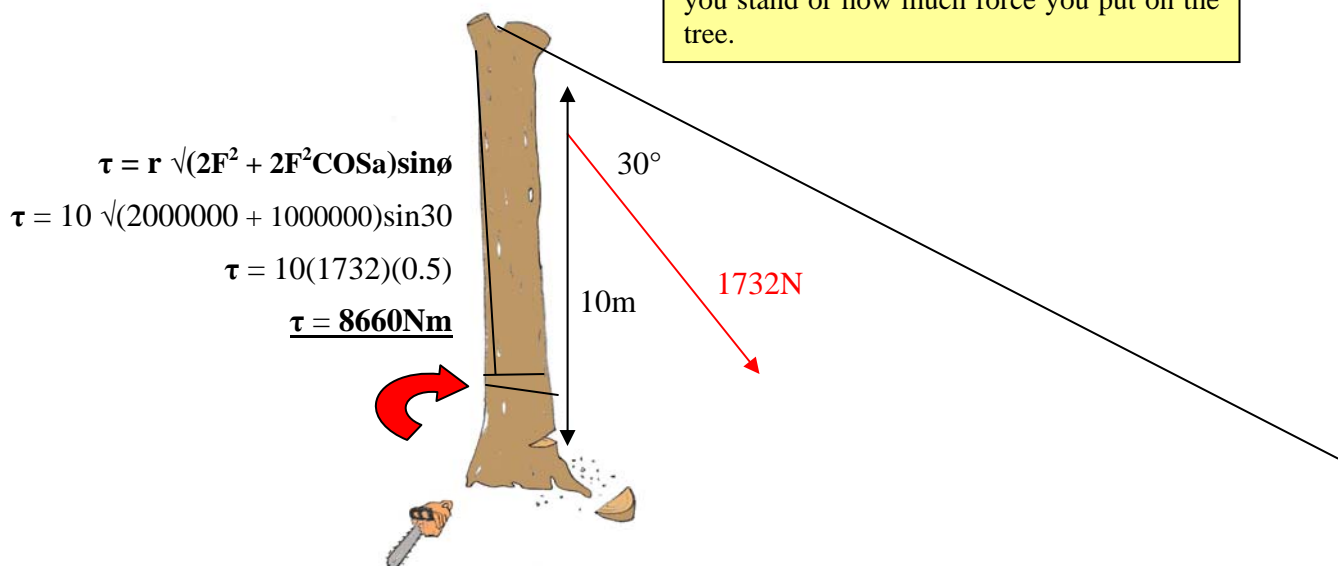
An interesting factor that has not been taken into account in the above examples is the reduction in turning moment on the tree caused by the reduced leverage of the head as it changes from being held out at 45° to being held hanging down. Try holding a shovel out flat at arm’s length and then hold it hanging down to see how dramatic this change can be. In the example shown above, **this reduction in turning moment would be enough to cause the tree to ‘sit up’ quite dramatically** – take care that you are expecting this result!

3.2 PULLING TREES OVER

Many tree workers will be familiar with the two different methods of tying a pulling line to a tree shown below. In both examples, the angle at which the rope is being pulled and the force being placed upon it is fixed; let us say because it is tied to a winch which is attached to another tree. In the first picture the rope has been tied off at the top in the more usual manner; in the second picture the rope has been passed over the fork then tied off at the base.



Interestingly enough, the torque applied at the hinge using both methods is exactly the same, and stays the same no matter where you stand or how much force you put on the tree.



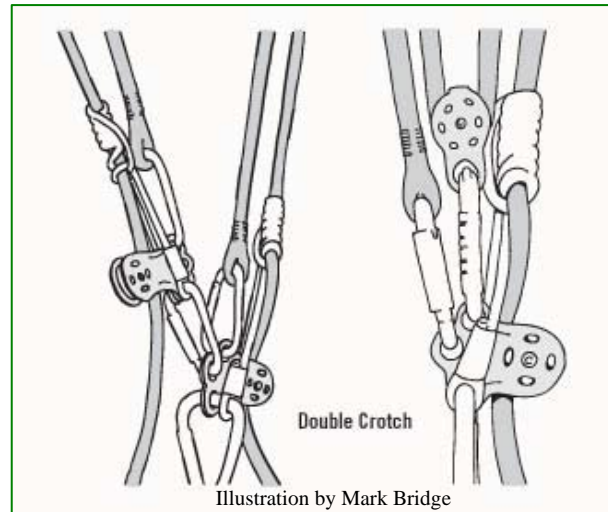
3.3 DOUBLE CROTCHING AND HIGH LINING

Many climbers will be familiar with the use of multiple or distributed high points. The V-Rig and M-Rig pictured below are both ways for a climber to smoothly and quickly take advantage of the many benefits of having two high points. In addition, **it is possible to create a high point or rigging point directly over a tree by setting a pulley on a line rigged between two larger neighbours. (High-Lining)**

These are powerful techniques which offer great benefits when applied in the correct situation. There is however an equally powerful risk inherent in any climbing technique which suspends a load between two high points.

Very large forces can be generated whilst double-crotching, specifically when the climber traverses from one crotch to another whilst level with the anchor points, or when the suspension line has a small angle of deflection.

The same risk applies, and is even easier to generate, when high-lining. In this situation, as the 'high-line' is installed and tensioned prior to the load being applied, it is particularly hard to predict the forces involved.



Using the M-Rig

Setups like this which exploit the enormous versatility of the hitch-climber system are extremely strong techniques for movement in the canopy. Interested climbers should check out [the hitch-climbers guide to the canopy](#), available on the Treemagineers website and elsewhere.

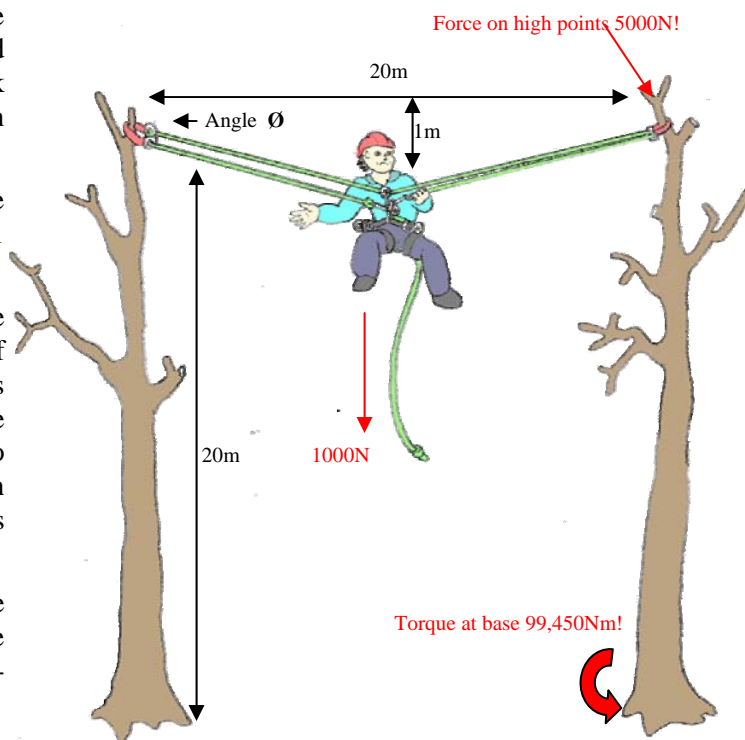
In the unlikely (and not to scale) illustration to the right, a 100kg apprentice has been sent to try and traverse from tree to tree, 20 metres up in the air. He has built an M-Rig for the purpose, which he installed from the ground before ascending up the right-hand tree. Once level with his high point, he took all of the slack out of his system, then began the traverse.

At the precise mid-point of his traverse, he is 10 metres from either tree, and only 1 metre below his high points.

As we can calculate using the formulae given below, he is generating a great deal of force on either high point by using this technique. In addition, the vector of the force generated has an inclination close to perpendicular to the angle of the tree, which means (remember the torque law) that he is generating a lot of torque.

Note that although the situation shown here uses an M-Rig, the same forces are generated much more easily whilst high-lining.

A graph of force generated against angle of deflection has been given in **Appendix I: Magnitude Graphs** on page 26.



Double Crotching and High Lining

Where a load **f** is suspended between two points at equal height, equidistant from either point, the magnitude of the force **F** on each point is given by

$$F = \frac{f}{2\sin\theta}$$

Where θ is the angle made between the rope and an imaginary horizontal line between the two points. This can also be written as:

$$F = \frac{f(\sqrt{x^2 + y^2})}{2x}$$

Where **x** is the vertical distance that the rope has deflected below the horizontal and **y** is the horizontal distance of the load from each point.

4 SUMMARY



Let us look again at the picture to the left, and see if we can model in more detail and with better accuracy the forces that were involved.

We know from the [work of Andreas Detter and associates](#) that the peak load on the rigging line is greatest when the branch is being swung in toward the trunk, approximately at the point shown in the picture.

Have a look at the rigging rope angles.

On the left hand stem, the rope makes a wide (obtuse) angle across

the rigging pulley, and is therefore generating a very low net force on the pulley. In addition, the force vector acts almost perfectly along the stem as compression force. At this point, the climber would feel the stem sitting up as the torque has been reduced. Although the stem will begin to experience an increase in force as the snatched head swings below it, in fact the net torque generated will have an increasingly 'upward' (clockwise from this perspective) tendency, though always of a small magnitude as the force vector on the pulley remains close to parallel with the stem.

On the right hand stem the situation is not so good. The angle is much more acute, and the stem has a significant bend. At the lower end of the bend (see arrow on picture) the net force generated on the pulley is generating a significant upward (anticlockwise) torque.

The forces at work on this stem could have been significantly improved by changing the rope angle across the (right-hand) pulley. In this case, that means changing the angle of the rope below the pulley, as it heads toward the friction device at the base of the tree. Assuming that there was nowhere else to site the friction device, the angle could be adjusted by using a pulley and another rope to redirect the line.

But is it all worthwhile, when the climber in the picture above could simply have gone further out and taken a smaller piece? These techniques often involve working with complex forces of large magnitudes, and climbers should ensure that they have a thorough understanding of the forces involved before any application.

In the right situation however, and used appropriately, an understanding of the principles above can make that nightmare tree job into 'just another day in the office'.

5 APPENDIX I: MAGNITUDE GRAPHS

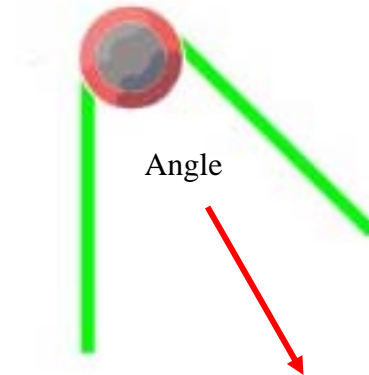
5.1: GRAPH OF THE COSINE LAW:

THE COSINE LAW:

The net force **F** on a point will bisect the angle made by the rope at that point, such that:

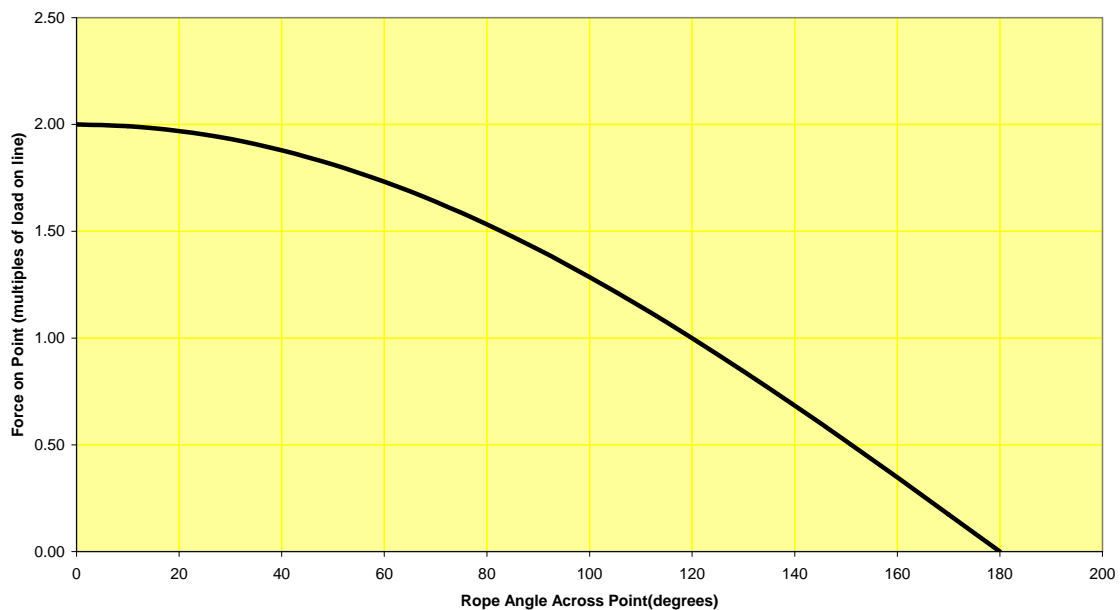
$$F^2 = 2X^2 + 2X^2\cos a$$

Where **X** is the force on the rope, and **a** is the angle made by the rope across the point.



Magnitude of Force (given as multiple of force on rope)

Graph showing Force of Rope on Point at Varying Angle



5.2: GRAPH OF THE TORQUE LAW:

THE TORQUE LAW:

$$\tau = r F \sin\theta$$

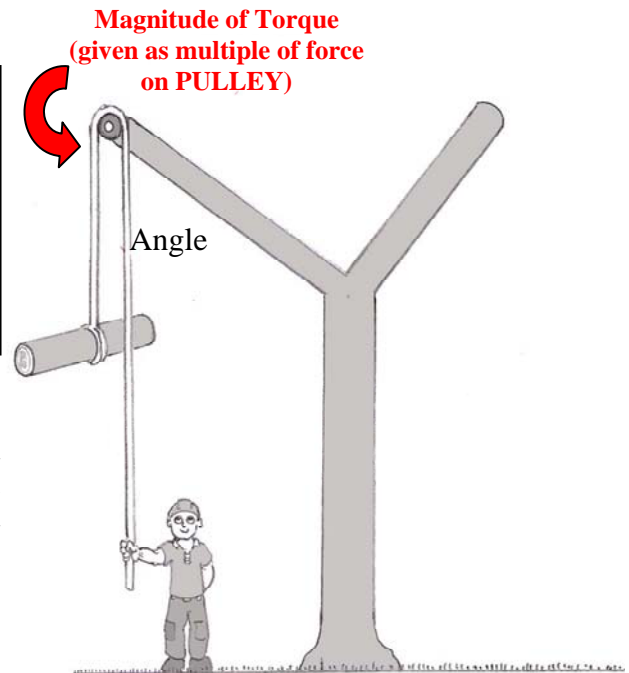
τ is the torque applied at the axis

r is the distance from the axis to the force

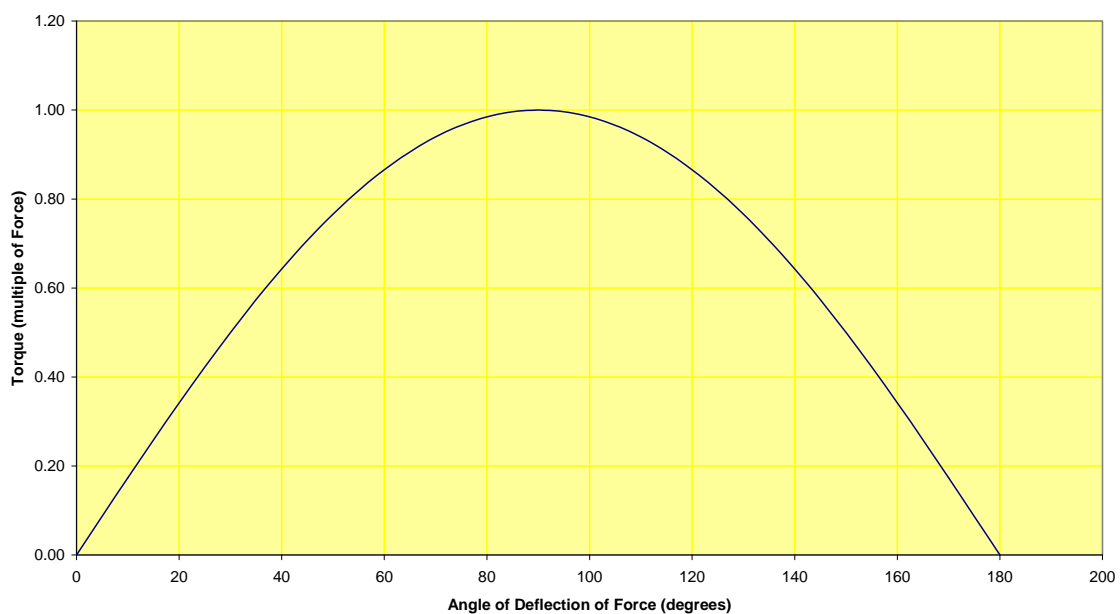
F is the magnitude of the force being applied

θ is the angle between the force and the arm

Note that this graph does not include the length of the lever arm r . To calculate the magnitude of torque on a union you need to multiply the result found on the graph below by the distance from the union to the pulley.



Graph of Angle of Deflection of Force against Torque Applied by Force



5.3: GRAPH OF THE FORCES OF DOUBLE-CROTCHING OR HIGH LINING

Double Crotching and High Lining

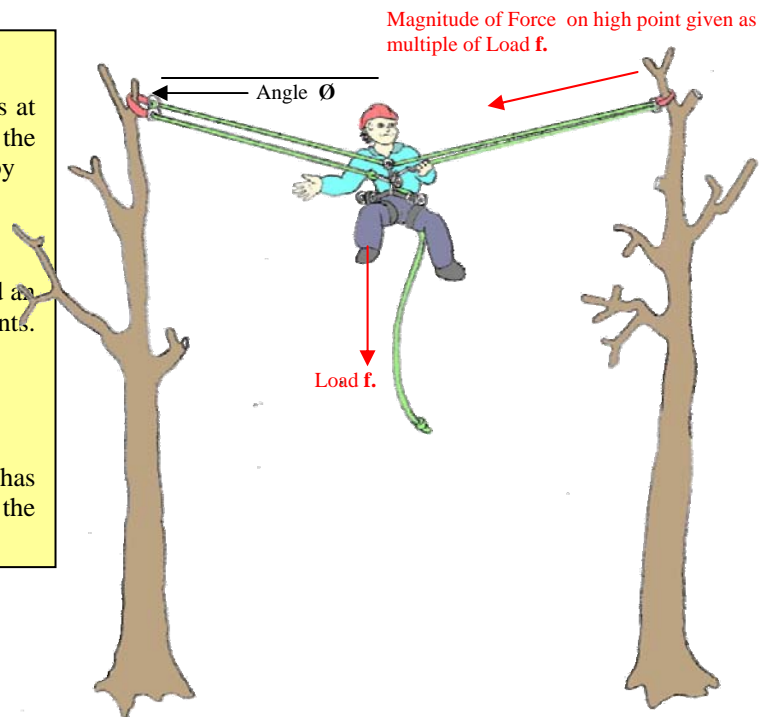
Where a load **f** is suspended between two points at equal height, equidistant from either point, the magnitude of the force **F** on each point is given by

$$F = \frac{f}{2\sin\theta}$$

Where θ is the angle made between the rope and an imaginary horizontal line between the two points. This can also be written as:

$$F = \frac{f(\sqrt{x^2 + y^2})}{2x}$$

Where **x** is the vertical distance that the rope has deflected below the horizontal and **y** is the horizontal distance of the load from each point.



Graph of Double-Crotching and High Line Forces

